

Econ 172A, W2002: Midterm Examination II

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has 3 questions. Answer them all.
3. If you do not know how to interpret a question, then ask me.
4. You must justify your answers to the first question. For each part, provide a brief explanation of how you came up with your answer suffices. No justification is required for the second and third questions. Please read and answer these questions carefully. Each part of questions 2 and 3 will be graded on a “right” or “wrong” basis – no partial credit will be awarded.
5. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		66
II		35
III		39
Exam Total		140

1. On the last homework assignment I asked you to formulate a version of the following problem:

Professor Foster’s family owns 125 acres of land (“Foster Farms”) and has \$40,000 in funds available to invest in the farm. The family works on the farm. They can supply a total of 3,500 person-hours of labor during the winter months and 4,000 person-hours during the summer. Any labor that is not used farming can be used to do odd jobs (like teaching Econ 120). These jobs pay \$5.00 per hour during the winter and \$6.00 per hour during the summer. Cash income comes from three crops, soybeans, corn, and oats, and two types of livestock, cows and hens. No investments are needed for the crops. On the other hand, cows require an investment of \$1,200 each, while hens cost \$9 each. Each cow requires 1.5 acres of land, 100 person-hours of work during the winter and 50 hours of person-hours during the summer. Each cow produces an annual cash income of \$1,000. Similarly, for each hen the farm earns \$5, requires no acreage, but uses .6 person-hours during the winter and an additional .3 hours during the summer. The farm has a chicken house that has capacity of 3,000 hens and a barn that can hold up to 32 cows. The table below provides relevant information on the input requirements and profitability of the crops. The Foster family wishes to determine how much acreage should be planted in each of the crops and how many cows and hens should be kept to maximize its net cash income.

	Soybeans	Corn	Oats
Winter person-hours	20	35	10
Summer person-hours	50	75	40
Net annual cash income (\$)	500	750	350

I came up with the following formulation:

I found it useful to define three kinds of variable: $x_S, x_C,$ and x_O are the number of acres used for soybeans, corn, and oats, respectively. y_H and y_C are the number of hens and cows. l_W and l_S are the number of unused hours of labor in winter and summer. With these definitions, the problem becomes:

$$\begin{array}{rcll}
 \max & 1000y_C & + & 5y_H & + & 500x_S & + & 750x_C & + & 350x_O & + & 5l_W & + & 6l_S & & \\
 \text{subject to} & 1.5y_C & & & & x_S & + & x_C & + & x_O & & & & & & \leq & 125 \\
 & 1200y_C & + & 9y_H & & & & & & & & & & & & \leq & 40000 \\
 & 100y_C & + & .6y_H & + & 20x_S & + & 35x_C & + & 10x_O & + & l_W & & & & \leq & 3500 \\
 & 50y_C & + & .3y_H & + & 50x_S & + & 75x_C & + & 40x_O & & & + & l_S & \leq & 4000 \\
 & y_C & & & & & & & & & & & & & & \leq & 32 \\
 & & & y_H & & & & & & & & & & & & \leq & 3000 \\
 & y_C & & y_H & & x_S & & x_C & & x_O & & l_W & & l_S & \geq & 0
 \end{array}$$

The constraints are, in order, land, investment, winter labor, summer labor, barn capacity, chicken house capacity, and nonnegativity. Note that the formulation assumes that the \$40,000 investment budget can only be used on the farm (that is, the objective function does not include unused investment funds). Note also that fractional values for the solution are permitted (but not plausible) – so do not worry if the solution involves a fractional cow or hen.

I solved the problem using Excel and attach the relevant Excel output. Use this information to answer the following questions as completely as possible. You should not resolve the linear programming problem, but you should state when it is necessary to resolve the problem to obtain a complete solution. Answer each part independently of other parts.

- (a) What is the solution to Professor Foster's problem and how much does he earn?
- (b) Professor Peters offers to buy 20 acres of Foster's land for \$20 per acre. Should Professor Foster accept the offer? Why or why not?
- (c) Professor Foster's son Andrew spends \$10,000 of the investment funds on skateboard equipment (leaving only \$30,000 to invest). How does this change the production plan and profits?
- (d) Andrew's skills as a skeleton racer permit him to earn \$7 per hour during the winter as an instructor (raising the payment of surplus winter labor to \$7 per hour). How does this change the production plan and profits?
- (e) Suppose that it is possible to grow artichokes on Professor Foster's farm. For each acre of land devoted to growing artichokes, he must invest \$100, use 30 person-hours of winter labor and 40 person-hours of summer labor. What is the minimum net annual cash income from artichokes needed to make it profitable for Foster to grow this crop?
- (f) Foster decides to quit his job teaching economics and devote more time to his farm. This increases the amount of winter labor available by 500 person hours (but does not change the amount of summer labor available). How does this change the production plan and profits?
- (g) How high would the price of corn need to be before it is profitable for Foster to grow corn?
- (h) Hogs require an investment of \$1,000 each, require 10 acres of land, and require 100 person-hours of labor in both winter and summer. Would Foster raise hogs if he they produced an annual cash income of \$1,200 each? Why or why not?
- (i) Foster's cows gain so much weight that they start taking up more space in the barn. Suppose that the barn can only hold 20 cows. What can you say about Foster's production plan and profits?
- (j) How would the solution and value to Professor Foster's problem change if he were required to raise at least one hen on the farm?

2. Consider the following (imaginary) description of my bread-baking problem: I can make three different kinds of bread. A loaf of whole wheat bread uses one pound of whole wheat flour and an ounce of yeast. A loaf of oatmeal-rye bread uses one pound of white flour, one quarter pound of rye flour, one quarter pound of oatmeal, and an ounce of yeast. A loaf of white bread uses three quarters of a pound of white flour and two ounces of yeast. I can sell a loaf of whole wheat bread for \$2.00, a loaf of oatmeal-rye bread for \$2.50, and a loaf of white bread for \$1.50. Each day I have available 120 pounds of whole wheat flour, 100 pounds of white flour, 50 pounds of rye flour, 30 pounds of oatmeal, and 140 ounces of yeast. In addition, my ovens are able to bake at most 125 loaves each day. I want to know how many loaves of each type of bread to produce in order to maximize profits subject to the constraints above.

Define the following variables:

x_{wheat} = the number of loaves of whole wheat bread produced.

x_O = the number of loaves of oatmeal-rye bread produced.

x_{white} = the number of loaves of white bread produced.

y_{wheat} = the number of pounds of wheat flour used.

y_{white} = the number of pounds of white flour used.

y_O = the number of pounds of oatmeal used.

y_r = the number of pounds of rye flour used.

y_y = the number of ounces of yeast used.

- (a) Which of the following constraints is appropriate for the problem? If a constraint is appropriate, explain how it relates to the problem description above. If a constraint is not appropriate, explain what the constraint says and why it does not apply to the problem.

i. $y_{white} = x_O + \frac{3}{4}x_{white}$

ii. $x_{white} = 2y_y + \frac{3}{4}y_{white}$

iii. $y_y \leq 140$

- (b) Using the variable definitions above, write an expression for the objective function of this problem.
- (c) Using the variable definitions above, write a linear constraint that guarantees that at least 50% percent of the loaves produced are white bread.
- (d) Using the variable definitions above, write a linear constraint that guarantees that at least as much white flour is used in the production of white bread as in the production of oat-rye bread.

3. Suppose that you solve a pair of linear programming problems, a primal, (P):

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and its dual, (D),

$$\max b \cdot y \text{ subject to } yA \geq c, y \geq 0.$$

Assume that both problems have at least two constraints (in addition to the non-negativity constraints), x^* is the solution to (P), and y^* is the solution to (D).

Decide whether each statement below is always true, sometimes true, or never true (under the stated conditions of the problem). That is, for each statement below, write “always,” if the statement is true; “sometimes,” if there exists problems (P) and (D) above, with solutions x^* and y^* , such that the statement is true **AND** there exist (different) problems and solutions such that the statement is false; and write “never” if the statement is never true.

- (a) Assume that each component of x^* is positive, $y_1^* > 0$, but all of the other components of $y^* = 0$.
- The first constraint in the primal is binding.
 - The second constraint in the primal is binding.
 - The first constraint in the dual is binding.
 - The second constraint in the dual is not binding.
 - The value of the primal is $b_1 y_1^*$.
 - $x_1 y_1 > 0$.
- (b) Assume that the odd components of x^* (x_1, x_3, \dots) are positive; the even components of x^* (x_2, x_4, \dots) are zero; the first two constraints of (P) are binding, the other constraints of (P) are not binding.
- The odd constraints of (P) are binding.
 - At most two of the components of x^* are positive.
 - Exactly two of the components of x^* are positive.
 - The first constraint of the dual is binding.
 - The second constraint of the dual is binding.
 - $y_1^* > 0$.
 - The odd components of y are zero.