Econ 172A, W2002: Homework 3 Answers

1. In order to solve this problem you need to create an additional job, this job absorbs the excess supply of labor. In the first three parts, this extra job brings no additional profit. In the third job, it adds to profit. I used Excel and obtained answers.

   The solution to parts a and c is:

	Project 1	Project 2	Project 3	Extra
Auditor 1	0	0	160	0
Auditor 2	0	110	0	50
Auditor 3	130	30	0	0

   The profit is \$72,900 in (a) and \$145,800 in (c). You could have predicted in advance that (c) would have the same solution as (a) (with profit doubled). All that doubling every charge does is "inflate" the currency. The problem is the same, the auditors just get twice as much money for doing the job. The solution to (b) is different. Notice that doubling only one worker's charges really does change the problem (while doubling all of the charges at the same time really does not). The solution to (b) is:

	Project 1	Project 2	Project 3	Extra
Auditor 1	0	0	160	0
Auditor 2	20	140	0	0
Auditor 3	110	0	0	50

   and the company earns \$87,200. The first assignment also solves (d), where the profits are \$75,900.

2. This is a standard assignment problem. You need to realize that you are assigning departure dates from San Diego to departure dates from Madison. Then you must price the tickets and solve.

   This table enters the appropriate airfares.

	Nov. 5	Nov. 11	Nov. 19	Nov. 25
Nov. 1	500	400	350	325
Nov. 9	400	500	400	350
Nov. 15	400	400	500	400
Nov. 24	350	350	400	500

   After subtracting constants from rows and columns to get a zero in each row and in each column we get:

	Nov. 5	Nov. 11	Nov. 19	Nov. 25
Nov. 1	175	75	**0**	0
Nov. 9	50	150	25	**0**
Nov. 15	**0**	0	75	0
Nov. 24	0	**0**	25	150

This table does admit a zero-cost schedule (there are actually several; one is in bold type. The solution I arrived at involves buying three round trip tickets for \$350 and one for \$400. The dates on the tickets would be: Nov. 1-19; Nov. 9-25; Nov. 5-15; and Nov. 11-24. (The first two round trips begin in San Diego; the last two begin in Madison.) Total cost: \$1,450.

For the second part, you must decide where the visits are going to take place. There are six possibilities (which two of the visits are going to be in San Diego) and these possibilities constrain the kinds of tickets you can buy (assuming that my wife can't travel on a ticket make out in my name). For example, if the first two visits are in San Diego, then you cannot buy a ticket for a stay longer than 10 days. So the question is, is it possible to buy tickets that cost \$1,450. The answer is: sure. The schedule described above works: assume that I buy the first and third round trips (so that we spend 1-5 and 15-19 in Madison) and my wife buys the other two trips (so we spend 9-11 and 24-25 in San Diego).

3. First I solved the minimization problem. I subtracted constants from each row to obtain:

	1	2	3	4	5	6
A	2	1	0	2	4	104
B	1	0	2	4	4	105
C	6	3	7	8	0	118
D	1	3	0	4	5	72
E	5	1	5	3	0	121
F	36	0	2	4	1	106

doing the same thing for the columns yields:

	1	2	3	4	5	6
A	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{4}$	$\underline{32}$
B	$\underline{0}$	$\vert\underline{0}$	$\vert 2$	$\vert 2$	$\vert 4$	$\vert 33$
C	5	$\vert 3$	7	6	$\vert 0$	46
D	$\vert\underline{0}$	$\vert\underline{3}$	$\underline{0}$	$\underline{2}$	$\vert\underline{5}$	$\underline{0}$
E	4	$\vert 1$	5	1	$\vert 0$	49
F	35	$\vert 0$	2	2	$\vert 1$	34

I cannot obtain a zero-cost assignment from this table, so I need to reduce costs. I crossed out columns 2 and 5 and rows A, B, and C to get rid of 0s. The minimum uncrossed number is 1. I subtract this from everything, then add it back to all rows and columns with lines (to preserve nonnegativity). I obtain:

	1	2	3	4	5	6
A	1	2	0	0	5	32
B	0	1	2	2	5	33
C	4	3	6	5	0	45
D	0	4	0	2	6	0
E	3	1	4	0	0	48
F	34	0	1	1	1	33

Now there is a zero-cost assignment. $A-3, B-1, C-5, D-6, E-4, F-2$. This agrees with Excel and costs 100.

To solve a maximization problem, you can convert to a minimization problem by multiplying the numbers in the original table by -1. You obtain:

	1	2	3	4	5	6
A	-8	-7	-6	-8	-10	-110
B	-6	-5	-7	-9	-9	-110
C	-8	-5	-9	-10	-2	-120
D	-4	-6	-3	-7	-4	-125
E	-9	-5	-9	-7	-4	-125
F	-8	-4	-6	-8	-5	-110

Now **add** the smallest number (that is, largest in absolute value, here 125) in each row to get a non-negative table:

	1	2	3	4	5	6
A	117	118	119	117	115	15
B	119	120	118	116	116	15
C	117	120	116	115	113	5
D	121	119	122	118	121	0
E	116	120	116	118	121	0
F	117	121	119	117	120	15

Now you can solve this problem like using the algorithm. Subtract from rows A, B, E, and F and columns 1-5 to get:

	1	2	3	4	5	6
A	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$
B	$\underline{2}$	$\underline{2}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{0}$
C	10	12	8	9	8	0
D	19	6	19	17	11	0
E	14	7	13	17	11	0
F	$\underline{0}$	$\underline{3}$	$\underline{1}$	$\underline{1}$	$\underline{5}$	$\underline{0}$

	1	2	3	4	5	6
A	$\underline{0}$	$\vert\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\vert\underline{6}$
B	$\underline{2}$	$\vert\underline{2}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\vert\underline{6}$
C	4	$\vert 6$	2	3	2	$\vert 0$
D	13	$\vert 0$	13	11	5	$\vert 0$
E	8	$\vert 1$	7	11	5	$\vert 0$
F	$\underline{0}$	$\vert\underline{3}$	$\underline{1}$	$\underline{1}$	$\underline{5}$	$\vert\underline{6}$

	1	2	3	4	5	6
A	0	2	1	1	0	8
B	2	4	0	0	1	8
C	2	6	0	1	0	0
D	11	0	11	9	3	0
E	6	1	5	9	3	0
F	0	5	1	1	5	8

This array tells you the solution: the answer is to match $A-5; B-4; C-3; D-2; E-6; F-1$ and yields the value 167.