Econ 172A, Winter 2002: Problem Set 2, Suggested Answers

Comments:

1. I have posted these separately.

2. (a) I found it useful to define three kinds of variable: x_S, x_C, and x_O are the number of acres used for soybeans, corn, and oats, respectively. y_H and y_C are the number of hens and cows. l_W and l_S are the number of unused hours of labor in winter and summer. Of course, you can denote all the variables by x_i. Also, you do not need to have separate variables for the surplus labor. I just found that using these variables makes the problem and the dual more transparent. With there definitions, the problem becomes:

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\begin{array}{lrcrcrcrcrcrcrcrcr}
\max & 1000y_C & + & 5y_H & + & 500x_S & + & 750x_C & + & 350x_O & + & 5l_W & + & 6l_S & & \\
\text{subject to} & 1.5y_C & & & + & x_S & + & x_C & + & x_O & & & & & \le & 125 \\
& 1200y_C & + & 9y_H & & & & & & & & & & & \le & 40000 \\
& 100y_C & + & .6y_H & + & 20x_S & + & 35x_C & + & 10x_O & + & l_W & & & \le & 3500 \\
& 50y_C & + & .3y_H & + & 50x_S & + & 75x_C & + & 40x_O & & & + & l_S & \le & 4000 \\
& y_C & & & & & & & & & & & & & \le & 3000 \\
& & & y_H & & & & & & & & & & & \le & 32 \\
& y_C & & y_H & & x_S & & x_C & & x_O & & l_W & & l_S & \ge & 0
\end{array}
$$

The constraints are, in order, land, investment, winter labor, summer labor, barn capacity, chicken house capacity, and nonnegativity. I think that the only possible confusion is the way that I introduced the variables l_W and l_S. These are added onto the left-hand sides of the labor constraints and also appear in the objective function. Notice that even though I wrote the labor constraints as inequalities, the constraints must bind when we solve the problem. The Foster's won't "throw away" labor that they could "sell" for at least \$5 per hour. Two other things to note. In my formulation I assumed that hens require .3 hours of labor in the winter. You could interpret the problem as stating that hens require $.6 + .3 = .9$ hours of labor in winter (that is, .3 *additional* hours. This change influences the answer to the problem, but it is a reasonable interpretation of the problem description. Finally, I have assumed that there is no value to having left over investment money. Alternatively, you might assume that any money not invested should be included in the objective function.

(b)

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\begin{array}{lrcrcrcrcrcrcr}
\min & 125z_1 & + & 40000z_2 & + & 3500z_3 & + & 4000z_4 & + & 3000z_5 & + & 32z_6 & & \\
\text{subject to} & 1.5z_1 & + & 1200z_2 & + & 100z_3 & + & 50z_4 & + & z_5 & & & \ge & 1000 \\
& & & 9z_2 & + & .6z_3 & + & .3z_4 & & & + & z_6 & \ge & 5 \\
& z_1 & & & + & 20z_3 & + & 50z_4 & & & & & \ge & 500 \\
& z_1 & & & + & 35z_3 & + & 75z_4 & & & & & \ge & 750 \\
& z_1 & & & + & 10z_3 & + & 40z_4 & & & & & \ge & 350 \\
& & & & & z_3 & & & & & & & \ge & 5 \\
& & & & & & & z_4 & & & & & \ge & 6 \\
& z_1 & & z_2 & & z_3 & & z_4 & & z_5 & & z_6 & \ge & 0
\end{array}
$$

(c) It is convenient to treat this as a production problem. The dual variables are the prices of inputs. For example, z_1 the dual variable associated with the first constraint is the value of an additional acre of land to the Foster family farm. The constraints of the dual guarantee that it is at least as profitable to sell the land at the dual prices than to operate one of the many enterprises (raising cows, planting soy) that are possible.

(d) Solution of Primal (via Excel): $y_C = 23.75, x_S = 56.25$, and profit 51875. The shadow price (dual variable) associated with winter labor is equal to \$6.25 and the price of summer labor is \$7.50. All other dual variables are zero. The value is 51875.

(e) You should check five kinds of thing. First, the value of the primal and dual are the same. Second, if a primal variable is positive, then the associated dual constraint must bind. Since there are two positive primal variables (y_C and x_S), check that the first and third constraints in the dual bind. Third, if a primal constraint is not binding, then the associated dual variable must be zero. The first, second, fifth, and sixth primal constraints don't bind (slack is positive). As should be the case, the associated dual variables are zero. Fourth, if a dual variable is positive, then the associated primal constraint binds. The third and fourth dual variables are positive and, indeed, these constraints bind in the primal. Finally, if a dual constraint is not binding (like all but the first and third), the associated primal variable must be zero. And they are.

3. (a) You produce 133.33 cans of each mixture and make a profit of \$440.

(b) This change is within the allowable range, so the solution stays the same. Profit goes up by 10 cents for each cheap mixture sold, so it is now \$453.33.

(c) This change is outside of the allowable range (the coefficient goes down by 40 cents, which is greater than the allowable decrease of 39.09 cents). You know that profit will fall, but you don't know the details unless you solve the problem again. I solved the problem again using Excel and I got: produce 41.67 cans of the cheap mixture and 333.33 cans of the party mixture (and no deluxe). Profit is now \$387.50. (Check that if you didn't change your production plan profits would be \$386.67.)

(d) This change is within the allowable range, the shadow price of peanuts is 1.02, so profit falls by this much times 20, or profit falls by \$20.44 (to \$419.56).

(e) This change is within the allowable range, the shadow price is \$1.91 per pound, so the extra almonds are worth \$19.10.

(f) Almonds and cashews are both worth \$1.91 per pound, to make a twelve ounce can of alshews you need \$1.91 times .75 (12 ounces is three quarters of a pound) worth of material, so a bit more than \$1.43 per can is the break even point.

(g) The original solution does not satisfy this extra constraint, so we must resolve the problem. The solution (on Excel) is: 150.9259259, 100, and 146.2962963 cans of cheap, party, and deluxe mixes respectively. Profit is now: \$436 and some change.

(h) The original solution satisfies this extra constraint, so the solution (and value) does not change.

(i) There are several ways to approach this problem. One method is to introduce 3 new variables, y_i is the amount of miracle nut used to replace ingredient i. The sum of all of the y_i is the amount of miracle nut that you use. This means that the new problem has three new, non-negative variables; the sum of these variables must be no more than 100 pounds; and that the right-hand sides of the original constraints go by y_i. If the allowable increase of the right-hand side variable with the largest shadow price was at least 100, then you could solve the problem without additional computation. (If you had just 15 pounds of miracle nut, then you would use it instead of almonds. In fact, since the price of almonds and cashews is equal, you know that the first 28.289 pounds of the new nut will replace cashews and almonds. After that, however, your production plan changes. So I used Excel. I learned that the solution involves making 155.5555556, 0, 377.7777778 cans of cheap, party, and deluxe mix respectively and using the miracle nut as a substitute for 65 pounds of cashews and 35 pounds of almonds. Profit becomes $631.11. Notice that profit goes up by $191.11, which is 100 times the shadow price of the most valuable resource. I was not able to predict that from the sensitivity and answer reports.