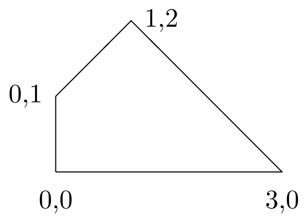
Econ 172A, W2002, Possible Answers

Comments Here are possible answers to the first problem set.

1. (a) Here is a picture. (The feasible set is the figure and its interior.)



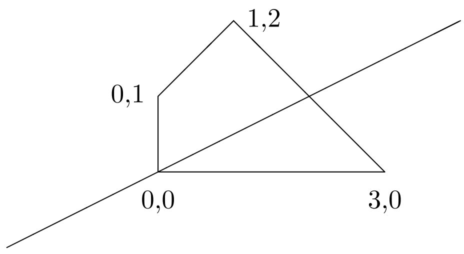
(b) i. $x^* = (3, 0)$, value $= 3$.

ii. $x^* =$ segment connecting $(3, 0)$ to $(1, 2)$, value $= 3$.

iii. $x^* = (3, 0)$, value $= 3$.

iv. $x^* = (1, 2)$, value $= 3$.

In these answers, x^* is the name that I give to the solution. What follows is the picture with a line in which $x_1 - 2x_2$ is constant. Shifting the line parallel, down, and to the right increases the value, hence tells you that the solution to (iii) is $(3, 0)$. Shifting the line parallel, up, and to the left decreases the value and (since $\max x_0$ is the same as $\min -x_0$) provides an answer to (iv). The other two parts are similar.



The corners of the feasible set (apparent from the picture) are: $(0,0)$, $(1,0)$, $(3,0)$, and $(1,2)$. From (b), (i) gives an objective function with unique solution at $(3,0)$ and (iv) gives an objective function with unique solution at $(1,2)$. For $(0,0)$ one possibility is $x_0 = -x_1 - x_2$; for $(1,0)$, one possibility is $x_0 = -10x_1 + x_2$. (The idea is to play around with slopes of objective functions that make different corners solutions. There are many possible solutions.)

(c) The Excel spreadsheet contains the template for the problem. The answers are the same as the graphical answers except that Excel does not indicate multiple solutions (the particular solution the Excel finds for you will depend on how you entered the data).

(d) Write the initial array (remembering to add slack variables, and to use row 0 to correctly write the objective function: note the signs of the coefficients).

Row	*Basis*	x_1	x_2	x_3	x_4	*Value*
(0)	x_0	1	-2	0	0	0
(1)	x_3	1	1	1	0	3
(2)	x_4	-1	$< 1 >$	0	1	1

You must pivot x_2 into the basis (only negative number in row 0) and x_4 out of the basis (minimum ratio rule).

Row	*Basis*	x_1	x_2	x_3	x_4	*Value*
(0)	x_0	-1	0	0	2	2
(1)	x_3	$< 2 >$	0	1	-1	2
(2)	x_2	-1	1	0	1	1

Again, there is a unique place to pivot.

Row	*Basis*	x_1	x_2	x_3	x_4	*Value*
(0)	x_0	0	0	.5	1.5	3
(1)	x_1	1	0	.5	$-.5$	1
(2)	x_2	0	1	.5	.5	2

Now Row 0 is nonnegative, so we have a solution. We can read it off and see that it agrees with the solution found earlier.

(e) There is no need to do additional computations. This change doesn't change the solution. It multiplies values by 5. (All you are doing is changing units.)

(f) This change does absolutely nothing. The answers and values stay the same. Perhaps the quickest way to see this is to graph the "new" feasible set: It is exactly the same as the old feasible set.

(g) What happens here is that the units of x_2 only are changed. The problem would be exactly the same if I created a new variable, called it y_2, set $y_2 = 5x_2$, and replaced x_2 by y_2 everywhere in the problem. Hence the values don't change, the x_1 part of the solution doesn't change; the x_2 part is multiplied by .2. (So, for example, the solution to part b, (iv) is $(1, .4)$.)

(h) This change really changes the problem. For the new (f), it changes the sense of the optimization. The new solutions are

i. $x^* =$ segment connecting $(0, 0)$ to $(0, 1)$, value $= 0$.
ii. $x^* = (0, 0)$, value $= 3$.
iii. $x^* = (1, 2)$, value $= 15$.
iv. $x^* = (3, 0)$, value $= 15$.

For the new (g), you change the direction of the second constraint. The new feasible set is unbounded: it is bounded by the ray starting at $(0, 1)$ and going through $(0, 2)$ and the ray starting at $(0, 1)$ and going through $(1, 2)$. Part (iii) has a unique solution at $(0, 1)$, with value -2. The other three parts are unbounded. (You can tell this by graphing or using Excel.)

2. The difference between this problem and the original one is that now x_1 is unconstained. That is, x_1 can take on negative values. The feasible set is now the triangle with vertices $(-1, 0)$, $(1, 2)$, and $(3, 0)$. The solutions don't change in any part, but this is because I didn't choose the right x_0. Can you think of an x_0 that would lead this problem to have a different answer than problem 1?

To write the problem in the first form, just replace x_1 by $u_1 - v_1$ and assume that the new variables are nonnegative.

$$\begin{array}{lccccccc}
\text{max} & u_1 & - & v_1 & & & & \\
\text{subject to} & u_1 & - & v_1 & + & x_2 & \leq & 3 \\
 & -u_1 & & +v_1 & + & x_2 & \leq & 1 \\
 & u_1 & & v_1 & & x_2 & \geq & 0
\end{array}$$

For the second part I must also include non-negative slack variables:

$$\begin{array}{lccccccccccc}
\text{max} & u_1 & - & v_1 & & & & & & & & \\
\text{subject to} & u_1 & - & v_1 & + & x_2 & + & s_1 & & & = & 3 \\
 & -u_1 & & +v_1 & + & x_2 & & & + & s_2 & = & 1 \\
 & u_1 & & v_1 & & x_2 & & s_1 & & s_2 & \geq & 0
\end{array}$$

3. This problem differs from the first one because it omits the first constraint. The resulting feasible set is unbounded. Reflection could (should?) tell you that solutions will either stay the same or values will go up. (When you make the feasible set larger, you won't do worse.) In fact, the problem is unbounded for all of the objective functions, as you can see either from Excel or from graphing.