Econ 172A, Winter 2002: Problem Set 1

Instructions: Due: January 31, 2002, in class (no late papers). Please supply complete answers. Unless otherwise noted on homework assignments and on examinations, you are required to explain how you got your answer. Simply stating a numerical answer is insufficient. For this assignment, attach excel spreadsheets when relevant and indicate the answer (and the question). This assignment asks you to solve many linear programming problems, but most are variations on the same basic problem. Set up one template for the excel computations and then make simple changes to get answers for different problems. You need not include a separate printout for every simplex computation as long as you provide a clear descriptions of how you got the answers. You are responsible for figuring out how to get Excel answers yourself (that is, I won't lecture on it). The notes on Excel should be sufficient. For this assignment there is no need to provide answer reports and sensitivity reports, but do indicate which cells on your spreadsheet have the solution. For graphs, clearly label the graph and show where the objective function is and how you identified a solution. For simplex algorithm computations, explain briefly how you selected pivot operations and what led you to stop. If I ask you to solve a problem, then tell me the solution (the best x) and the value (x_0 evaluated for the best choice of x).

1. Consider the linear programming problem:

   Find x to solve:

$$\begin{array}{lrcrcl} \max & x_0 & & & & \\ \text{subject to} & x_1 & + & x_2 & \leq & 3 \\ & -x_1 & + & x_2 & \leq & 1 \\ & & & x & \geq & 0 \end{array}$$

   In this problem, x_0 is the objective function.

   (a) Graph the feasible set of the linear programming problem.

   (b) Solve the problem graphically for each of the following objective functions:

      i. $x_0 = x_1$.

      ii. $x_0 = x_1 + x_2$.

      iii. $x_0 = x_1 - 2x_2$.

      iv. $x_0 = -x_1 + 2x_2$.

   (c) Identify the corners of the feasible set. For each corner, give an example of an objective function, x_0, such that the solution of the linear programming problem for that x_0 occurs at that corner (and only at that corner). (So you need a different x_0 for each corner.)

   (d) Solve each of the problems in the previous part using Excel. Compare your answers. Are there any differences? Explain.

(e) Use the simplex algorithm to solve the problem with $x_0 = -x_1 + 2x_2$. Again note the differences (if any) between the solution you find and what you found in earlier parts of the problem. (If you want practice with the simplex algorithm, feel free to solve the other problems.)

(f) Multiply each of the objective functions in part (b) by 5. Solve the new problem (any method). How do the solutions and values change?

(g) Multiply the second constraint of the problem by 5 (so that it becomes $-5x_1 + 5x_2 \le 5$). Resolve the problem for the objective functions in part (b) (any method). How do the solutions and values change?

(h) Multiply the coefficient of x_2 in each constraint of the problem and in each of the objective functions in part (b) by 5. Resolve the problems. How do the solutions and values change?

(Note parts (e), (f), and (g) are independent. That is, for example, when you do part (f) do not multiply the objective function by 5.)

(i) Repeat parts (f) and (g), except this time multiply by -5 instead of 5.

If you think, then you should be able to do parts (f) through (h) with a minimum of computation. If you don't think, you should still be able to do these parts easily. When you get your answers, please compare them to earlier answers and comment on how they have (or have not) changed.

2. Consider the linear programming problem:

$$\begin{array}{lrcrcl} \text{max} & x_0 & & & & \\ \text{subject to} & x_1 & + & x_2 & \le & 3 \\ & -x_1 & + & x_2 & \le & 1 \\ & & & x_2 & \ge & 0 \end{array}$$

(a) Write the linear programming problem (with objective function as in question 1, part b, (i)) in the form:

i. $\max c \cdot x$ subject to $Ax \le b, x \ge 0$.

ii. $\max c \cdot x$ subject to $Ax = b, x \ge 0$.

(b) Solve the linear programming problem for the objective functions in problem 1 (b).

3. Repeat part (b) of the previous problem for the linear programming problem:

$$\begin{array}{lrcrcl} \text{max} & x_0 & & & & \\ \text{subject to} & -x_1 & + & x_2 & \le & 1 \\ & & & x & \ge & 0 \end{array}$$