ANSWER KEY

Test 1
Economics 136 – Human Resources
Fall 2012
Prof. Julian Betts

October 18, 2012

Name: _____

Student ID _____

There are 4 written problems in this test, worth a total of 51 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the graders.

If you use pencil, the exam cannot be regraded. If you do submit your test for regrading, you must do within the time and other guidelines listed in the syllabus.

SHOW ALL YOUR WORK!

You have 80 minutes. Good luck.

For the graders:

1.	_____/15
2.	_____/14
3.	_____/9
4.	_____/13
SUM	_____/51

1. (15 points) Your firm needs to hire 50 representatives, one for each state, to work with clients in those states. Apart from one 60-minute phone meeting per year these representatives do not work directly with each other in any way.

a) Which of the following better describes these 50 workers:

i) Workers work independently

ii) Workers are complementary to each other.

(CIRCLE ONE) (1)

b) Currently the market wages in the East half of U.S. for college graduates and high school graduates are $25 per hour and $12.50 per hour respectively. In the West half of the U.S. wages for college graduates and high school graduates are $25 per hour and $10 per hour respectively. Your analysis of past hires indicates that the value marginal product per hour of representatives who are college graduates to representatives is given by

$VMP_{college} = \$46$ and $VMP_{High\ School} = \$20$

These productivity levels do not vary by region.

Which type of workers should you hire as representatives in the East and the West? Why? Explain with an equation or two. (4)

East: $\left(\frac{C}{Y}\right)_{Highschool} = \frac{\$12.50}{\$20} = 0.625$ (1)

$\left(\frac{C}{Y}\right)_{college} = \frac{\$25}{\$46} = 0.54347$ (1)

Hire only college graduates in the East as cost/output ratio is lower (1/2)

West: only difference is wage of high school graduates

So $\left(\frac{C}{Y}\right)_{college} = 0.54347$ still b-t

$\left(\frac{C}{Y}\right)_{High\ school} = \frac{\$10}{\$20} = 0.5$ (1)

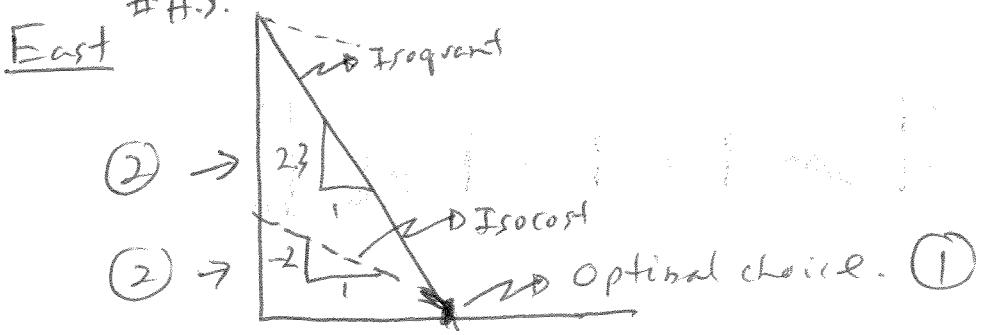
Hire only graduates as they now have lower cost to output ratio (1/2)

c) Illustrate by drawing a graph with a typical isoquant and isocost lines for the East. *Place the number of high school graduates on the vertical axis.* Write the slopes of the lines on the graph. Be sure to label the optimal choice of number of high school and college workers for a given isoquant.

Then draw a separate graph showing the corresponding information for the West.

(10)

East



Note: I did not ask students to derive the slopes so they don't need to do this. But if they do show work & make a mistake give them an appropriate # of points!

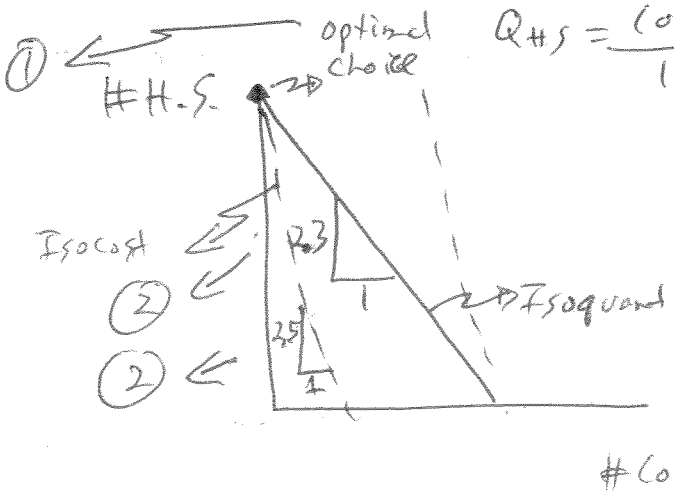
$Q = 46 Q_C + 20 Q_{HS}$ Isoquant: $Q_{HS} = \frac{Q}{20} - 2.3 Q_C$

Isocost: $25 Q_C + 12.50 Q_{HS} = Cost \Rightarrow$ $Q_{HS} = \frac{Cost}{12.5} - 2 Q_C$

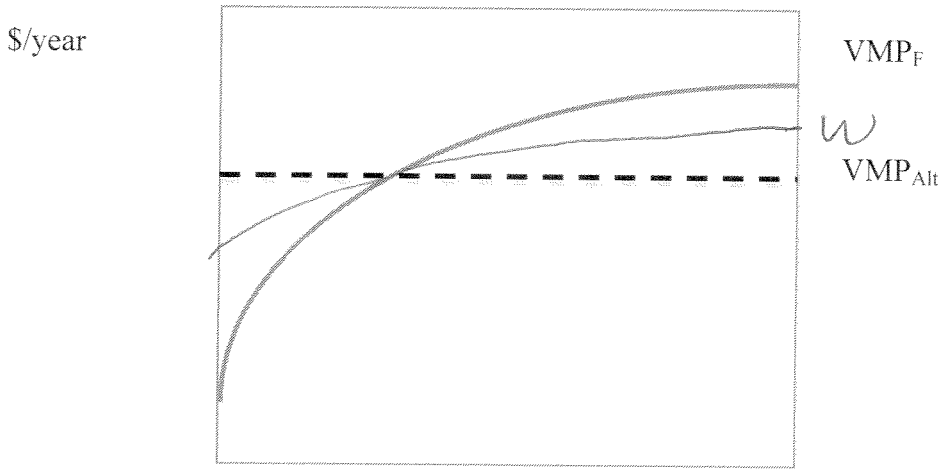
West Same Isoquant as above

Isocost $25 Q_C + 10 Q_{HS} = Cost \Rightarrow$

$Q_{HS} = \frac{Cost}{10} - 2.5 Q_C$



2) (14 points) Your firm provides on-the-job-training (OJT) that increases workers' productivity at the current firm but not at other firms. The graph below shows the VMP of the worker at your firm (VMP_F) and at other firms in the local labor market (VMP_{Alt}). The former graph assumes that the worker joins your firm at age 18 and potentially stays until age 65.



a) Is this OJT specific or general? (1 point)

Specific

b) Would it be optimal for your firm to pay the worker VMP_{Alt} in all periods? Explain. (3 points)

No. (1)
In later periods the worker could threaten to leave unless gets paid more than VMP_{Alt}. The firm would probably agree as productivity is much higher than VMP_{Alt}. Overall the firm would lose money (2)

c) Would it be optimal for your firm to pay the worker his or her actual productivity VMP_F in all periods? Explain. (3 points)

No. (1)
The worker should not accept because in later periods firm could threaten to ~~[illegible]~~ fire the worker unless worker accepts a lower wage (between VMP_F and VMP_{Alt}). The worker would probably agree as would still earn more than his or her VMP_{Alt} elsewhere. (2)

d) Go back to the diagram above and draw in a possible wage profile, which you can label W, which would make both the firm and the worker agree to provide/receive training. Then in the space below explain why your choice would make both firm and worker agree to provide/receive training. (5 points)

(1) for drawing W is between VMP_{FS} & VMP_{Alt} (see last page)

(2) Firm loses money in early years but makes a profit in later years so won't try to fire in later years.

(2) Worker earns less than alternative in early years but earns more than alternative in later years. Won't threaten to quit in later years.

e) Is the type of skill that your firm is providing to the worker better described as a type of skill for which there is a thin or thick market, or neither type of market? Explain in a sentence. (2 points)

(1) Thin

(1) Other firms don't value this training so is not a market for this training/skill outside firm.

3. (9 points) The theory of signaling states that education does not make people more productive, but that it allows more productive workers to signal their productivity to potential employers, who have no other means to learn workers' productivity.

a) Suppose that firms have decided that to stay in business they need to identify high productivity (type H) and low productivity workers (type L) and pay them accordingly. The actual productivity of type H workers is 8, and for type L workers, productivity is 5. Firms decide to pay workers a wage given by the amount of school S that workers receive, with higher salaries if schooling is at least equal to some level S* that the firms have chosen. This wage is:

$$W = \begin{cases} 8 \text{ if } S \geq S^* \\ 5 \text{ otherwise} \end{cases}$$

Both types of workers have a utility function:
U = W – Costs of education

But the costs of education are different between type H and L workers:
Costs of $\text{Education}_H = 2S$ and
Costs of $\text{Education}_L = 4S$

a) Derive S^*_{min} which is the value of S* firms could set that would make type L workers indifferent between obtaining that level of education and getting no education (S=0). *Hint for parts a) and b): don't panic if you don't get solutions that are integers!!* (3)

Set U for L workers equal:

$5 - 0 = 8 - 4S^*_{min}$ (2)

$S^*_{min} = \frac{3}{4}$ (1)

b) Now derive S^*_{max} which is the level of S* that would make type H workers indifferent between obtaining that level of education and obtaining no education (S=0). (3)

Set utility between $S=0$ & $S=S^*_{max}$ equal to each other:

$5 - 0 = 8 - 2S^*_{max}$ (2)

$S^*_{max} = \frac{3}{2}$ (1)

c) Based on your answers to a) and b) what is the range of S* values that firms could choose that would induce type H workers to obtain S* and type L workers to choose S=0. Is this range a set of separating equilibria or pooling equilibria? (3)

If set $\frac{3}{4} < S^* < \frac{3}{2}$ then L chooses $S=0$ & H chooses $S=S^*$ (2)

these are Separating equilibria (1)

Note: Give $1\frac{1}{2}$ out of (2) if they state as equality: $\frac{3}{4} \leq S^* \leq \frac{3}{2}$

4. (13 points) Your boss tells you to design a probationary wage system under which workers earn W_1 during period 1 and W_2 in period 2. Your goal is to make skilled workers want to work for your firm, and to prevent unskilled workers from working for your firm. To simplify the math, let's assume that your goal is to leave unskilled workers indifferent between applying and not applying. Assume that workers maximize the sum of earnings over 2 periods, and that their discount rate=0. Unskilled and skilled workers' wages in the general labor market are:
W_U and $W_S = W_U + G$.
After one period, you will keep all workers you believe to be skilled and fire all those you believe to be unskilled. However, there is a problem: there is a probability p that at the end of period 1 an unskilled worker will be identified as "skilled" and will not be fired. This probability obeys $0<p<1$.

a) Derive the values of W_1 and W_2 that will make skilled workers want to work at your firm, and which will leave less skilled workers indifferent between applying and not applying. Show your work and explain each step in words. (6)

To attract skilled workers

$W_1 + W_2 \geq 2W_S$ (O.K. if write W_S as $W_U + G$) (1)

Profit maximization/Cost minimization $\Rightarrow$

$W_1 + W_2 = 2W_S$ (i) (1)

To make unskilled workers indifferent between applying to this firm set w_1 & w_2 such that

$$w_1 + [pw_2 + (1-p)w_u] = 2w_u \quad \text{(ii)} \qquad (1)$$

Note: students will solve these 2 equations in various ways. Here is just one approach. Subtract (i) from ii

$$-[w_1 + w_2] + w_1 + [pw_2 + (1-p)w_u] = -2w_s + 2w_u$$

$$-(1-p)w_2 + (1-p)w_u = -2w_s + 2w_u$$

$$(-1-p)w_u + 2w_s = (1-p)w_2$$

$$w_2 = \frac{2w_s - (1+p)w_u}{1-p} \quad \text{(iii)} \qquad (2)$$

Note: Give partial points if make algebraic mistake

Substituting into (i) $\Rightarrow$

$$w_1 + \frac{2w_s - (1+p)w_u}{1-p} = 2w_s$$

$$w_1 = \frac{-2w_s + (1+p)w_u}{1-p} + 2w_s$$

$$w_1 = \frac{(1+p)w_u - 2pw_s}{1-p} \quad \text{(iv)} \qquad (1)$$

b) Did you find that you should set the wages differently in periods 1 and 2? Explain the intuition behind this result in a few sentences. (2 points)

Yes, Set $W_1 < W_2$ because unskilled workers get W_1 for sure but W_2 only with probability $P < 1$. So by ↓ W_1 and ↑ W_2 we can ↓ expected earnings for the unskilled applicants. (2)

c) What happens to W_1 and W_2 as p rises? Show your work and derive the derivatives. What is the intuition for your result? (5)

$$\frac{2W_1}{2P} = \frac{[W_u - 2W_s](1-P) - (-1)[(1+P)W_u - 2PW_s]}{(1-P)^2}$$

$$= \frac{W_u(1-P+1+P) + W_s[-2(1-P) - 2P]}{(1-P)^2}$$

$$\overset{\#}{=} \frac{-2(W_s - W_u)}{(1-P)^2} < 0 \qquad (2)$$

$$\frac{2W_2}{2P} = \frac{-W_u(1-P) - (-1)[2W_s - (1+P)W_u]}{(1-P)^2}$$

$$= \frac{2W_s + W_u[-1+P-1-P]}{(1-P)^2}$$

$$= \frac{2(W_s - W_u)}{(1-P)^2} > 0 \qquad (2)$$

Intuition: As P ↑ it increases the chances that unskilled workers will get W_2. This ↑ desire of unskilled workers to apply. To counteract this the firm will ↓ W_1 and ↑ W_2 by equal amounts. This lowers the expected earnings of unskilled workers at the firm because they receive W_1 with probability 1 and W_2 with probability $P < 1$. (1)