Answer Key

Final Exam
Economics 136 – Human Resources
Fall 2012
Prof. Julian Betts

December 11, 2012
Name: _____
Student ID _____
There are 6 written problems in this exam, worth a total of 66 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the graders.

If you use pencil, the exam cannot be regraded. If you do submit your exam for regrading, you must do within the time and other guidelines listed in the syllabus.

SHOW ALL YOUR WORK!

You have 3 hours. Good luck.

For the graders:

1. _____/26
2. _____/14
3. _____/4
4. _____/8
5. _____/4
6. _____/10

SUM _____/66

Please sign the following optional form if you would like us to be able to return this exam to you in a publicly accessible location.

OPTIONAL STUDENT CONSENT FOR RELEASE OF STUDENT INFORMATION
(Buckley Waiver)

I hereby authorize the UCSD Economics Department to return my graded final examination/research paper by placing it in a location accessible to all students in the course. I understand that the return of my examination/research paper as described above may result in disclosure of personally identifiable information, that is not public information as defined in UCSD PPM 160-2, and I hereby consent to the disclosure of such information.

Quarter Fall 20012 Course Econ 136 Date Dec. 11.

Instructor Julian Betts

Student ID#

Print Name

Signature

1. (26 points) **Before you do parts a, b, read through parts c and d. In answering questions c and d your answer will be much easier to write down if you carefully number your equations in parts a and b, so that you can refer to them later!**

At your company, the product you sell is priced at $16 each and sales are produced by worker i as follows:

$$q_i = m_i + e_i \qquad (1)$$

where q is the number of units sold, m is effort, and e is a luck factor with mean 0, reflecting a luck factor over which the worker has no control.

The costs to the firm are the wages it pays workers, plus $2 per widget in materials and energy costs.

There are two workers, j and k. Each worker experiences luck, e_j and e_k, affecting output of workers j and k respectively. $x = e_k - e_j$ takes on values between -Z and +Z, where Z is some positive number, with a uniform probability distribution.

Workers are risk neutral, so that their utility equals their expected wage minus the disutility of effort.

$$U = E(\text{Earnings}) - m_i^2 \qquad (2)$$

For workers to accept a job at your firm you must offer them a package where utility is at least the reservation utility, U_R, where $U_R \geq 0$.

a) Your firm decides to set up a piece rate system in which workers are paid $\alpha + \beta q_i$ where $q_i = m_i + e_i$. Because the workers are risk neutral, the expected utility to the worker of being paid this way is just the expected wage of $\alpha + \beta m_i$, minus the disutility of effort. Derive the profit-maximizing values of α and β, optimal worker effort, and expected utility for the worker and expected profits per worker for the firm. Show ALL of your work. (8)

(1) Worker j: $\max_{m_j} \alpha + \beta m_j - m_j^2$

(1) F.O.C. $\beta - 2m_j^* = 0$

(1/2) $\Rightarrow m_j^* = \beta/2$ (i)

S.O.C. $-2 < 0$ so U maximum

1½ (½ for each constraint & ½ for the function)

Firm: $\max_{\alpha,\beta} (16-2)m_j^* - \alpha - \beta m_j^*$ subject to (i) and $\alpha + \beta m_j = m_j^2 + U_R$ (ii)

Substitute (ii) $\Rightarrow$

Firm: $\max_{\beta} 14m_j^* - m_j^{*2} - U_r$ s.t. (i)

1½ → Deduct if take ROC w.r.t. m

F.O.C. $(14 - 2m_j^*)\frac{dm_j^*}{d\beta} = 0$ But (i) $\Rightarrow \frac{dm_j^*}{d\beta} = \frac{1}{2} \neq 0 \Rightarrow$

(1/2) $\rightarrow m_j^* = 7$ (iii)

Substitute (iii) into (i) $\Rightarrow$

$\beta^* = 14$ (iv) ½

Substitute (iii) + (iv) into (ii)

$\alpha^* = -\beta^* n_j^* + n_j^{*2} + U_r$

$\alpha^* = -14(7) + (7)^2 + U_r$

$\alpha^* = -49 + U_R$ (v) ½

$\pi/\text{worker} = 14 n_j^* - n_j^{*2} - U_r$

$= 14(7) - 7^2 - U_r$

$= 49 - U_R$ (vi) ½

$E(\text{Utility}) = \alpha^* + \beta^* n_j^* - n_j^{*2}$

$= -49 + U_R + 14(7) - 7^2$

$= U_R$ (vii) ½

b) Another way to induce effort is to have your two workers compete in a tournament. Suppose that after a trial period, one of two workers will be promoted to boss. The person with greater production will get the promotion. In this post-tournament period, the boss earns W_1 and the worker earns W_2, where $W_1 > W_2$.

Solve the workers' problems and the firm's problem. What are the optimal values of W_1 and W_2, and expected profit per worker and expected utility per worker? (Hint: You should find that $E(U) = U_R$!)

DEFINE ALL NEW VARIABLES THAT YOU USE THAT ARE NOT MENTIONED IN THIS QUESTION. (8 points)

Worker j

$$\max_{m_j} \; pW_1 + (1-p)W_2 - m_j^2$$ (1)

where p = Probability j wins tournament

$= \text{Prob}(m_j + e_j > m_k + e_k)$

$= \text{Prob}(e_k - e_j < m_j - m_k)$

$= G(m_j - m_k)$

where G is cumulative distribution function of $x = e_k - e_j$.

F.O.C.

$$(W_1 - W_2)\frac{\partial p}{\partial m_j} - 2m_j^* = 0 \qquad \text{(viii)}$$ (1)

$$\frac{\partial p}{\partial m_j} = g(m_j^* - m_k^*)$$

where g is p.d.f. for uniform density.

By symmetry $m_j^* = m_k^* \Rightarrow$

$$\frac{\partial p}{\partial m_j} = g(0)$$

$$g(a) = \begin{cases} \frac{1}{2z} & \text{if } -z \le a \le +z \\ 0 & \text{otherwise} \end{cases}$$

So $\frac{\partial p}{\partial m_j} = \frac{1}{2z}$

Substitute into (viii) $\Rightarrow$

(ix) $m_j^* = \frac{1}{4z}(W_1 - W_2)$. Now, let $m^* = m_j^* = m_k^*$ (1)

(1) Firm $\max_{W_1, W_2} 14(2m) - W_1 - W_2$ subject to (ix) and $E(U) = pW_1 + (1-p)W_2 - m^2 = U_R$

By symmetry $m_j^* = m_k^* \Rightarrow p = \frac{1}{2}$ $\quad \frac{W_1 + W_2}{2} = m^2 + U_R$ (x)

Subst (x) $\Rightarrow$ Firm $\max_{W_1, W_2}$ $28m^* - 2m^{*2} - 2U_R$

F.O.C. W_1: $(28 - 4m^*)\frac{\partial m^*}{2W_1} = 0$ and W_2: $(28 - 4m^*)\frac{\partial m^*}{2W_2} = 0$

$\frac{\partial m^*}{2W_1} \neq 0$ and $\frac{\partial m^*}{2W_2} \neq 0 \Rightarrow$ $m^* = 7$ (xi) (1/2)

(ix) $\Rightarrow W_1 - W_2 = 28z$ (x) $\Rightarrow W_1 + W_2 = 98 + 2U_R$

& subst $W_1 = W_2 + 28z$ into ↑

½ But if take F.O.C. w.r.t m

Subst. into $W_1 = W_2 + 28Z \Rightarrow$ $W_1 = 49 + 14Z + U_R$

$E(\pi/\text{worker}) = 14(7) - 7^2 - U_R = 49 - U_R$

(xii) ½

(xiii) ½

(xiv) ½

c) Does this method of payment lead to different profits per worker than the piece rate scheme you studied in part a? (2)

$E(U) = \frac{W_1 + W_2}{2} - e^2 = 49 + U_R - 49 = U_R$ (xv) ½

No

d) In the above question we specified that . $x = e_k - e_j$ takes on values between -Z and +Z, where Z is some positive number, with a uniform probability distribution. Note that the higher is Z, the more risky the tournament is. Calculate dW_1/dZ, and dW_2/dZ. Similarly, calculate what happens to average earnings, by calculating $d\{(W_1+W_2)/2\}dZ$. What is the intuition behind your result? (4)

$2W_1/2Z = 14$ (1)

$2W_2/2Z = -14$ (1)

$\frac{2\,\frac{W_1+W_2}{2}}{2Z} = 0$ (1)

Intuition: the larger is Z, the riskier the tournament, so to elicit π-maximizing effort, the firm must Δ the prize for winning = $W_1 - W_2$.
But average wage does not change as firm still just needs to set $E(U) = U_R$. (1)

e) How do the optimal piece rate (α and β) you calculated in a) and the optimal tournament wages W_1 and W_2 you calculated in b) change as the reservation utility U_R, rises? In each case, how does the worker's expected utility change with a one-unit increase in U_R? (4)

Piece rate

(iv) + (v) $\Rightarrow$

If U_R Δ by 1, α Δ by 1 $\frac{\partial \alpha^*}{\partial U_R} = 1$ but $\frac{\partial \beta^*}{\partial U_R} = 0$ (1)

$\frac{\partial E(U)}{\partial U_R} = 1$ from (vii) (1)

Tournament

(xii) + (xiii) $\Rightarrow$ $\frac{\partial W_1}{\partial U_R} = 1$ and $\frac{\partial W_2}{\partial U_R} = 1$ (1)

(xv) $\Rightarrow$ $\frac{\partial E(U)}{\partial U_R} = 1$ (1)

2.(14) A company has designed a stock option program that will align the interests of the Chief Executive Officer (CEO) of the company with the interests of shareholders. The problem is that the Board of Directors cannot directly monitor the CEO's effort, but it does know that a large increase in effort by the CEO could increase the profits of the company, and therefore the stock price of the firm.

The table below shows that regardless of whether there is an economic recession or boom, if the CEO puts in high effort the firm's profits and stock price will be higher than if he or she puts in low effort.

	Low Effort			High Effort		
State of economy	Probability	Profits	Stock price	Probability	Profits	Stock price
Recession	2/3	2 million	15	2/3	3 million	20
Boom	1/3	4 million	24	1/3	8 million	39

a) The firm offers the CEO base salary of $1 million plus 1 million call options that expire next year, with a strike price of $21. So expected earnings of the CEO are:
E(Earnings) = $1 million + E(value of stock options).
Calculate the expected earnings under the assumption that the CEO exerts low effort, E(Earnings|Low Effort). Also calculate expected earnings if the CEO exerts high effort, E(Earnings|High Effort) (4)

E(earnings | low effort) = $1 million + 1,000,000 $(\frac{1}{3})$ (24−21) (1)
= $2 million ← "1/3" (1)
E(earnings | high effort) = $1 million + 1,000,000 $(\frac{1}{3})$ (39−21) (1)
= $7 million (1)

b) The CEO's utility function is
U = E(Earnings) – C(Effort) where the latter term, the cost of effort, if given by

C(Effort) = 0 if Effort=Low and C(Effort)=$2 million if Effort=High

E(U|Low) = $2 million − 0 = $2 million (2)
E(U|High) = $7 million − $2 million = $5 million (2)

Calculate the utility of the CEO under the assumption of Low effort, and then recalculate utility under the assumption of High effort. Which level of effort will the CEO exert, high or low? Explain. (6)

See last page.

Picks High effort. (2)

c) In real life companies often grant employees stock options that are not vested, meaning that the options cannot be used by the employee until 4 or even 5 years into the future. What is the strategic reason why firms often issue call options to workers in this way? (4)

This induces workers to stay at the firm for a reasonable period of time (4)

Be generous with wording.

3.(4 points) Suppose that a firm has n tiers, where tier 1 is the lowest paid tier, tier 2 is the next lowest tier, while tier n represents the President of the company. In firms with hierarchies it has often been observed that the pay raise workers get for being promoted from the bottom tier (tier 1) to the next tier (tier 2) is smaller than the pay raise for getting promoted from tier 2 to tier 3, and so on. List two reasons why firms often make pay raises much higher for promotions at the upper tiers than the lower tiers.

1) It becomes more difficult to earn promotions the higher one goes "up the ladder". So must ↑ the pay raise to compensate workers competing for the promotion.

(2)

2) Part of incentive to work harder in lower tiers is prospect of promotion to higher tiers later in career.

(2)

4. (8 points) a) Explain in a sentence or two what the free rider problem is that arises when workers are compensated in part based on the production generated by the "team" to which they belong. (2)

Workers don't exert much effort because they receive only a fraction of the increase in firm revenues they would generate

(2)

b) In spite of this free rider problem, in the real world we do see firms paying workers at least in part based on how successful their team is. So there must be some good reasons

to do so. Briefly explain any three of the four main reasons we discussed in this course for why firms may want to pay workers at least in part as a function of the production of the team to which they belong. (6) Full points for ANY 3 of the following 4:

1) Workers are complements rather than substitutes
2) Free rider problems not major, perhaps because team is small or easy for workers to monitor others' effort
3) Gains from specialization
4) Gains from knowledge transfer

5. (4 points) Suppose a firm is trying to decide between having more young workers or older workers. In terms of skills, what are two advantages the older worker is likely to have? What are the two advantages of younger workers?

Older workers have:
1) more firm-specific human capital (1)
2) more On-the-job training (1)

Younger workers have:
1) more recent education (1)
2) more technological know-how (1)