

Test 1
Economics 136 – Human Resources
Winter 2005
Prof. Julian Betts

January 25, 2005

Name: _____

Student ID _____

There are 4 written problems in this exam, worth a total of 50 points. Please write neatly. If you place the answer to a question in an odd place, such as the back of the page, please indicate this clearly, for the sake of the graders.

If you use pencil, the exam cannot be regraded. If you do submit your test for regrading, you must do within the time and other guidelines listed in the syllabus.

SHOW ALL YOUR WORK!

You have 80 minutes. Good luck.

For the graders:

1.	_____	/16
2.	_____	/12
3.	_____	/10
4.	_____	/12
SUM	_____	/50

1. (16 points) Suppose that a firm has a production function given by $Q=L^{0.5}H^{0.5}$ where Q is output per hour and L and H are the numbers of workers who are low-skilled and high-skilled respectively.

a) Given this production function, at the firm in question do workers work independently of each other or in an interdependent fashion? Explain in a sentence. (2 points)

b) Calculate the marginal product of workers of either type, e.g. dQ/dL and dQ/dH . (2 points)

c) Suppose that there is no capital cost in this business, so that the total cost of hiring one worker with low or high skills is given by the corresponding wage rates W_L and W_H . Write down the Lagrangean and take the first-order conditions. Then use the first order conditions to calculate an equation for the cost-minimizing ratio of workers of each type, that is, L/H . (7 points)

d) Your company has signed a contract to produce 120 units of output per period to a client. Currently highly skilled workers earn \$14.40 per period and less skilled workers earn \$10 per period. What is the number of high skill and low skill workers, H and L, that your firm needs to hire? (2 points)

e) Draw a graph that shows the shape of the isoquant and isocost line, and illustrate the optimal combination of L and H for the given production level in problem d. You do not need to derive the general equation for the isoquant. Instead just show the general shape. For the isocost, do state the formula for the slope AND intercept. (3)

2) (12 points) Suppose that your profit-maximizing firm wants to hire only skilled workers, but it cannot detect a worker's skills until after the first period of employment. At the end of period 1, the firm will keep only the workers it believes are skilled workers. However, less skilled workers have a $\frac{1}{2}$ probability of escaping detection and being hired again in period 2. Workers can work for you at most **TWO** periods before retiring. In the general labor market, skilled and unskilled workers earn $W_S > W_U$ respectively per period. Workers maximize the sum of wages over the two periods of their working lives.

Suppose that the firm designs a probation program under which new hires earn a wage W_1 in period 1, and if they are not fired at the end of period 1 they earn W_2 in period 2.

a) Derive values of these two wages that will ensure that the skilled will want to work for your company while unskilled workers will be indifferent between working for your firm or another firm. Show your work. (10 points)

b) Prove using differentiation that as W_s rises the wage the firm must pay in period 1 must fall. In a sentence or two explain why this must be the case. (2 points)

3) (10 points) Suppose short people are just as productive, on average, as tall people. But members of both groups -- short and tall people -- vary in productivity. Suppose that it has been observed that short people do not get promoted as often to upper management positions as tall people, due to discrimination.

a) Is the average productivity of short people in upper management jobs higher, lower, or the same as the average productivity of tall people in upper management jobs? Why? (5)

b) Is the average productivity of short people in lower level jobs higher, lower, or the same as the average productivity of tall people in lower level jobs? Why? (5)

4) (12 points) Suppose that the labor force in your region consists of one-quarter each of workers with productivity of \$3, \$5, \$10 and \$12 per hour. At present there is only one type of firm, that pays a piece rate. Because it is costly to monitor output, these “piece rate” firms deduct \$2 per hour for the costs of monitoring. Thus, workers of the four types can earn the following per hour:

Worker type (productivity per hour)	Piece rate (output-\$2)/hour
\$3	\$1
\$5	\$3
\$10	\$8
\$12	\$10

Note that these firms earn zero profits per worker, although they will presumably earn the average return on their capital.

a) You have just graduated from university and started up your own company. You don't have either the time or the temperament to closely supervise your employees to measure their true productivity, so instead of paying piece rates like every other firm in the city, you decide to pay a fixed salary. You are very ambitious, so your goal is to hire as many local workers as you can, subject to making profits of zero or preferably higher. What fixed salary should you pay so as to attract as many local workers as possible? (Hint: if a

worker is currently paid, say, \$2.50, you can attract him with a wage or \$2.51 or higher.) Which workers will work at your fixed salary firm? Which will work at the piece rate firms? What will be your average profit per worker? (6 points)

b) Ignoring monitoring costs, at which firms are workers more productive on average? Explain. (2 points)

c) You foolishly brag about how well you are doing to all of your friends, many of whom have taken Economics 136. They decide to go into business as well, paying a fixed salary. Soon there are hundreds of fixed-salary firms just like yours, as well as the piece-rate firms that always existed. What will happen to the wage rate you and other fixed-salary firms end up paying? Which workers will work at the fixed-salary vs the piece rate firms now? Calculate the profit per hour per worker for the two types of firms. (4 points)