Econ 174 Financial Insurance
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UCSD

Final Exam

Please answer all four questions. Each question carries 25% of the total grade.

1. Explain the reasons why you **agree** or **disagree** with the following statements. Circle the correct answer and use the space to justify your choice.

(i) It is never optimal to exercise an American put option prior to its expiration date. /
Disagree, *because* **It is optimal to early exercise when $X-S_t > p_t$. In other words, it is optimal when the value of the stock is so low, that simply investing the value of the exercised put at the risk free rate is greater than the maximum expected payout.**

(ii) European put-call parity states that the cost of buying a call and a put option equals the price of the underlying stock.
Disagree, *because* $c_t - p_t = S_t - e^{-rt}(X)$

(iii) Implied volatility can be backed out of existing option prices. It shows how volatile the market expects future stock price movements to be.
Agree, *because* **Assuming the BS model is correct, we can find out what volatility would perfectly price the options at a given strike and maturity.**

(iv) An investor who has sold a call option and is hedging this position needs to buy more shares if the underlying stock price goes up.
Agree, *because* **The delta of a call goes to one as the stock price increases. Thus, the only way to hedge is to buy proportionally more shares.**

(v) The delta of a European put option lies between minus one and zero. It increases when the stock price goes up.
Agree, *because* **The delta of a European put is 0 when St>X and goes to -1 as St-> 0**

(vi) Pricing of options in the binomial tree only works if investors are risk-neutral.
Disagree, *because* **Pricing using a binomial tree works because we use no arbitrage pricing and is agnostic to the preferences for risk.**

2. The following binomial diagram gives a company's stock price at three points in time:

		144
	120	
100		96
	80	
		64
time 0	time 1	time 2

The risk-free interest rate is ln(1.06) per period. Each period is one year; *u* is 1.20 and *d* is 0.80. The stock pays no dividends prior to period 2.

(i) What is the risk-neutral probability?

p = e^{rT}-d/(u-d) = 1.06-.8/1.2-.8=.26/.4 =0.65

(ii) At each node in the tree, calculate the price of an *American call* with a strike price of 90 and expiration at time 2.

		C_{uu} =54
	C_u =.65(54)+.35(6)/1.06 = 35.09	
C =.65(35.09)+.35(3.68)=22.73		**C_{du} = 6**
	C_d = .65(6)+.35(0)/1.06 = 3.68	
		C_{dd} = 0
t=0	*t=1*	*t=2*

(iii) What would be the value of the call option at time 0, if it had been a European rather than an American call option?

c (European) = 22.73 It would be the same as the American call

(iv) At each node of the tree, compute the price of an *American put* option with a strike price of 100 and expiration at time 2.

		P_{uu} =0
	P_u =.65(0)+.35(4)/1.06=1.32	
P =.65(1.32)+.35(20.00)/1.06=7.41		**P_{du} = 4**
	P_d =.65(4)+.35(36)/1.06=14.33 (Early Exercise) $P_{d=}$20	
		P_{dd} = 36
t=0	*t=1*	*t=2*

(v) At the nodes at time 0 and 1, compute the American put option's delta

	Δ_u = (0-4)/(144-96)= -.0833
Δ =(1.32-20.00)/(120-80)= -.4670	
	Δ_d =(4-36)/(96-64)= -1
t = 0	*t = 1*

(v) Suppose you have sold a put option at time 0 and wish to set up a perfect hedge for the next period. How could you do this by trading the underlying stock? (give the relevant number of shares and explain if you would buy or sell the shares)

You would short sell .4670 shares of the underlying stock.

3. A company's reported stock prices over the last six trading days were as follows (moving from the earliest to the last price):

50 51 51 52 51 50

The stock price closed at 50. This volatility is typical of expected future volatility. Currently the continuously compounded interest rate is ln(1.06). The stock will not pay any dividends over the next 30 days.

Black-Scholes Formula

$c = S_0N(d_1) - Xe^{-rT}N(d_2)$, $p = Xe^{-rT}N(-d_2) - S_0N(-d_1)$,

$d_1 = [\ln(S_0/X)+(r+\sigma^2/2)T]/(\sigma\sqrt{T})$, $d_2 = d_1 - \sigma\sqrt{T}$.

(i) Use Black-Scholes to find the value of a European call option with 30 (trading) days to expiration and a strike price of 48.

Volatility estimate: σ = 31.11(annualized)
S0=50, X=48, T=30/252=.119, r=ln(1.06)=.0583

d_1 = 0.4986 **$N(d_1)$ =0.6910**
d_2 = 0.3912 **$N(d_2)$ = 0.6522**
BS call price = 3.46

(ii) What is the value of a European put option with 30 (trading) days to expiration and a strike price of 48?

$N(-d_2)$ =0.3478 **$N(-d_1)$ = 0.3090**
BS put price = 1.13

(iii) Compute the delta of the Black-Scholes call option

Call delta = 0.6910

(iv) Explain what this delta measures

This measures the change in the option price for a small change in the stock price. As the stock price increases, the delta goes to 1. When the stock price is far below X, it is near 0.

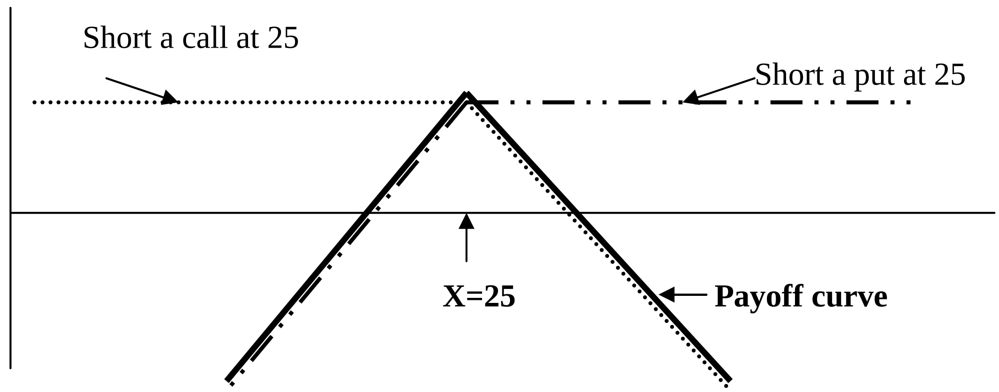
(iii) If the stock price, S_0, gets very large, what are the Black-Scholes values of the call and put options?

Call = $e^{rt}(S_0-X)$
Put = 0

4. On Friday November 17, 2000 Compaq's stock price closed at 25. The following option prices were quoted in WSJ:

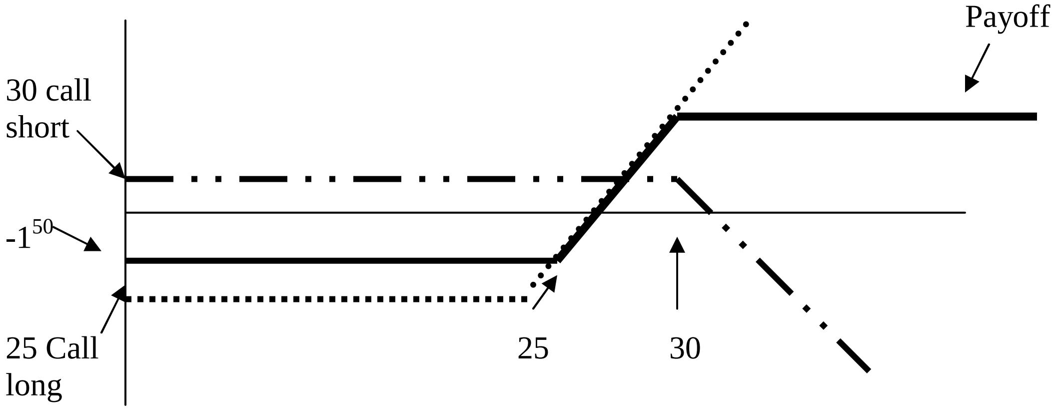
Strike	Expiration	Call Price	Put Price
22.50	Dec	3.30	0.90
25	Dec	1.85	1.80
27.50	Dec	0.85	3.20
30	Dec	0.35	5.40

(i) Plot the profit diagram for a *short* position in a December *straddle* with a strike price of 25 as a function of the stock price at expiration and fill in the requested information. **Short straddle involves shorting one call and one put at X=25.**



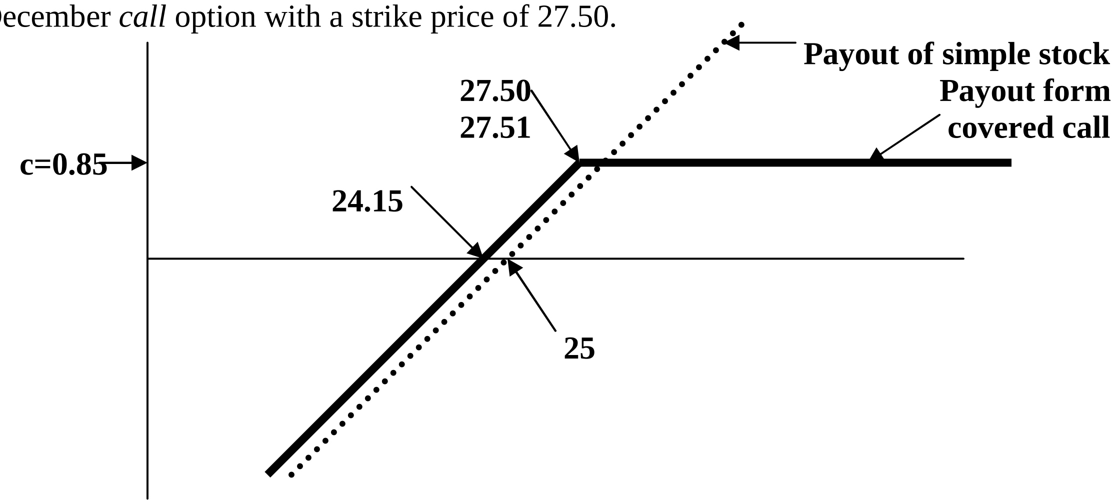
Maximum profit: 1.85+1.80 when S_t=25
Maximum loss: Unbounded as S_t->Infintiy

(ii) Plot the profit diagram (at expiration) for a *bull* spread based on the December call options with strike prices 25 and 30.



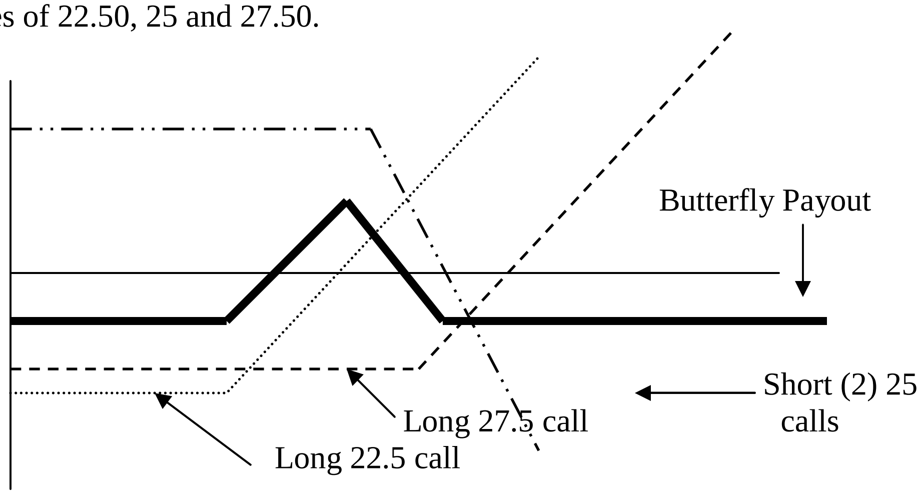
Range of stock prices at expiration with a positive profit: **St>26.50**

(iii) Show the profit (at expiration) from an investment that buys the *stock* and sells the December *call* option with a strike price of 27.50.



Maximum profit:2.5+.85 = 3.35
Break-even point: 24.15

(iv) Plot the profit diagram (at expiration) for a *butterfly* spread based on the call options with strike prices of 22.50, 25 and 27.50.



Maximum loss: -3.30+1.85+1.85-.85 =-.45 for St<22.5 or St>27.5
Maximum profit:-3.30+1.85+1.85-.85+2.50 =2.05 St=25